NAG Toolbox for MATLAB

f08cf

1 Purpose

f08cf generates all or part of the real m by m orthogonal matrix Q from a QL factorization computed by f08ce.

2 Syntax

```
[a, info] = f08cf(a, tau, 'm', m, 'n', n, 'k', k)
```

3 Description

f08cf is intended to be used after a call to f08ce, which performs a QL factorization of a real matrix A. The orthogonal matrix Q is represented as a product of elementary reflectors.

This function may be used to generate Q explicitly as a square matrix, or to form only its trailing columns.

Usually Q is determined from the QL factorization of an m by p matrix A with $m \geq p$. The whole of Q may be computed by:

```
[a, info] = f08cf(a, tau);
```

(note that the array **a** must have at least m columns) or its trailing p columns by:

```
[a, info] = f08cf(a(:,1:p), tau);
```

The columns of Q returned by the last call form an orthonormal basis for the space spanned by the columns of A; thus f08ce followed by f08cf can be used to orthogonalise the columns of A.

The information returned by f08ce also yields the QL factorization of the trailing k columns of A, where $k < p$. The orthogonal matrix arising from this factorization can be computed by:

```
[a, info] = f08cf(a, tau, 'k', k);
```

or its trailing k columns by:

```
[a, info] = f08cf(a(:,1:p), tau, 'k', k);
```

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda**,$*$) – **double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

Details of the vectors which define the elementary reflectors, as returned by f08ce.

2: **tau**$(*)$ **– double array**

Note: the dimension of the array **tau** must be at least $\max(1, \mathbf{k})$.

Further details of the elementary reflectors as returned by f08ce.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix Q.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix Q.

Constraint: $\mathbf{m} \geq \mathbf{n} \geq 0$.

3: **k – int32 scalar**

Default: The dimension of the array **tau**.

k, the number of elementary reflectors whose product defines the matrix Q.

Constraint: $\mathbf{n} \geq \mathbf{k} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a**(**lda,**$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix Q.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **k**, 4: **a**, 5: **lda**, 6: **tau**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $4mnk - 2(m+n)k^2 + \frac{4}{3}k^3$; when $n = k$, the number is approximately $\frac{2}{3}n^2(3m-n)$.

The complex analogue of this function is f08ct.

9 Example

```
a   =   [-0.2536561223671959,   -0.7751853859846198,   0.1094532397645297,
0.06960714430288747;
         -0.0956952802509345,  0.2275943648113135,  0.5121810603924651,  -
0.5958371552327169;
         -2.894845895288455,  0.2084279305583289,  -0.0952450839256265,  -
0.09745000202404246;
           -0.5041099118295458,  -1.581266529610948,  0.2476000680518517,
0.02227428617692399;
           1.921275206091976,  -1.053208461749375,  1.692758571755385,  -
0.5930528694606012;
          -0.8729501828758776,  0.9018026937371966,  0.213935545781127,  -
3.091585353827385];
tau = [1.863066450840819;
     1.179138368141079;
     1.487335114645626;
     1.16172932097152];
[aOut, info] = f08cf(a, tau)
```

```
aOut =
   -0.0833    0.9100   -0.2202   -0.0809
    0.2972   -0.1080   -0.2706    0.6922
   -0.6404   -0.2351    0.2220    0.1132
    0.4461   -0.1620   -0.3866   -0.0259
   -0.2938    0.2022    0.0015    0.6890
   -0.4575   -0.1946   -0.8243   -0.1617
info =
           0
```